Statistical Fatigue Life Prediction of a Damaged Structure Using Reduced Basis Digital twin

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Contents

Motivation

- Hydrogen distribution system using tube trailer
  - Exposed to various damage sources that can cause physical defects during the transportation

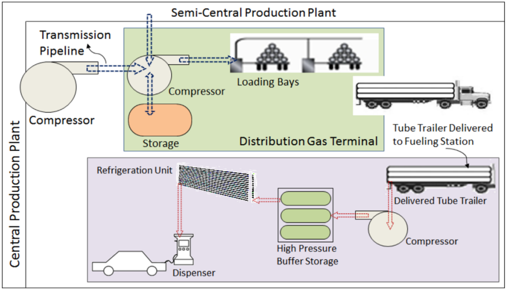


Figure 1: Storage, transportation, and charging process of a vessel†

- Predictive digital twin for structural health monitoring
  - Relieve safety concerns by keeping an eye on a high-pressure vessel
  - Prognose the status of a defect by updating a virtual model based on sensor data of a physical asset
  - Realize prognostic and health management (PHM) that improve safety and reduce operating costs at the same time

† Reddi, K., Mintz, M., Elgowainy, A., & Sutherland, E. (2016). Challenges and opportunities of hydrogen delivery via pipeline, tube-trailer, LIQUID tanker and methanation-natural gas grid. Hydrogen science and engineering: materials, processes, systems and technology, 849-874.

Motivation

- Previous research
  1. Digital twin based on a finite element (FE) model†
     - Accurate but computationally expensive for many-query problems

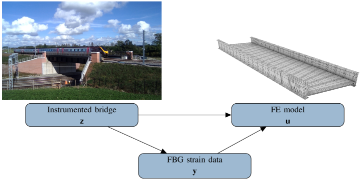


Figure 2: Digital twin of a bridge based on FE model†

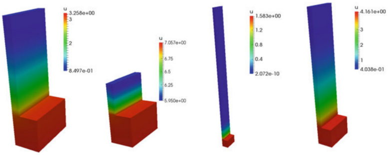


Figure 3: Reduced basis functions of a thermal fin problem††

  2. Digital twin based on a data-driven reduced-order modeling‡
     - Unable to guarantee the model follows governing physical rules
  3. Reduced basis (RB) method††: physics-driven reduced-order modeling
     - Achieve a significant reduction in computational time compared to the conventional FE method after the time-consuming offline phase
- Goal: digital twin-driven fatigue life prediction of a defected vessel using RB method

† Febrianto, E., Butler, L., Girolami, M., & Cirak, F. (2022). Digital twinning of self-sensing structures using the statistical finite element method. Data-Centric Engineering, 3, e31.
††Hesthaven, J. S., Rozza, G., & Stamm, B. (2016). Certified reduced basis methods for parametrized partial differential equations (Vol. 590). Berlin: Springer.
‡Fang, X., Wang, H., Li, W., Liu, G., & Cai, B. (2022). Fatigue crack growth prediction method for offshore platform based on digital twin. Ocean Engineering, 244, 110320.

Problem statement

- Statistical fatigue life prediction of a damaged vessel using digital twin
  - Consider uncertainties in system parameters E, ν, p, d
  - Estimate the fatigue life at the current status as the dent size grows
  - Assume that high fidelity FE data emulate the strain sensor data
  - Use the RB method to accelerate the simulation for the digital twin implementation

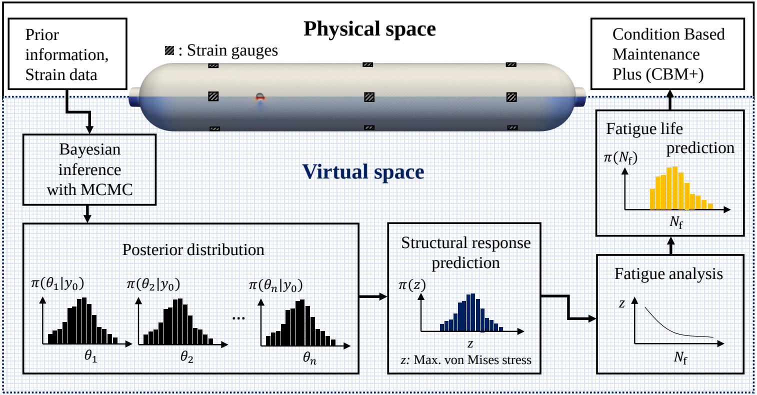


Figure 4: Overview of a statistical fatigue life prediction using digital twin

Methods

Bayesian inference

- Bayesian inference
  - Allow to estimate unknown system parameters with a probabilistic description
  - Estimate unknown system parameters by posterior parameter distribution $\pi(\theta|y_o)$ upon observation y_o

$$\pi(\theta|y_o) \propto \pi(y_o|\theta)\pi(\theta)$$

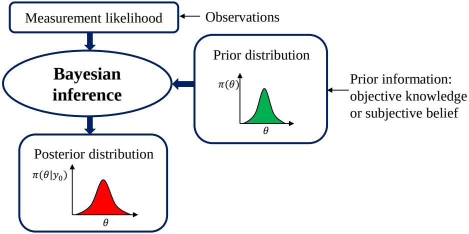


Figure 5: Concept of a Bayesian inference

- Markov-Chain Monte Carlo (MCMC) simulation†
  - Draw parameter samples in the form of a posterior probability distribution for unknown distributions
  - Use adaptive Metropolis within Gibbs sampling and Metropolized independent sampling successively

† Stark, P. B., & Tenorio, L. (2010). A primer of frequentist and Bayesian inference in inverse problems. Large-scale inverse problems and quantification of uncertainty, 9-32.

Methods

Fatigue analysis

- Follow procedures for fatigue analysis in ASME BPVC†
  - American Society of Mechanical Engineers Boiler and Pressure Vessel code
  - Provide the guidance to design and construction of pressure vessels for safe operation
- Steps for fatigue analysis
  1. Determine the load history of the vessel.
  2. Determine the individual cycles and define the total number of cyclic stress ranges in the load history.
  3. Determine the equivalent stress range for the cycle
  4. Determine the effective alternating equivalent stress amplitude for the cycle.
  5. Determine the number of cycles to failure for the alternating equivalent stress.

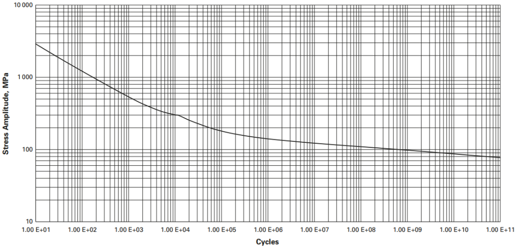


Figure 6: Fatigue curve of a vessel steel†

† The American Society of Mechanical Engineers, ASME Boiler & Pressure Vessel Code, Section VIII Division 2, 2019 Edition

Methods

Linear elasticity problem

- Strong form

$$\frac{\partial}{\partial x^{o}_{j}(\mu)}\left(C^{o}_{ijkl}(\mu)\frac{\partial u^{o}_{k}(\mu)}{\partial x^{o}_{l}(\mu)}\right)=0, \quad \text{in} \quad \Omega^{o}(\mu) \tag{1}$$

- Boundary conditions

$$u^{o}=0 \quad \text{on} \quad \Gamma^{o}_{1},\Gamma^{o}_{2}, \qquad C^{o}_{ijkl}\frac{\partial u^{o}_{k}}{\partial x^{o}_{l}}e_{\mathrm{n},j}=qe_{\mathrm{n},i} \quad \text{on} \quad \Gamma^{o}_{3} \tag{2}$$

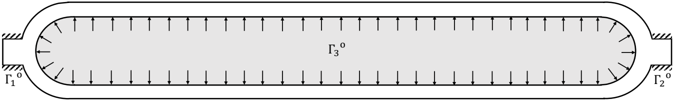


Figure 7: Boundary conditions of a damaged vessel model

- Compuational subdomains

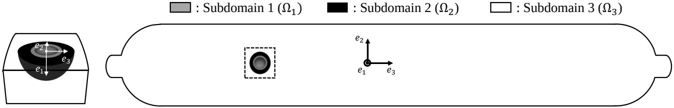


Figure 8: Computational subdomains of a damaged vessel

- Weak form

$$\sum_{s=1}^{3}\int_{\Omega^{o}_{s}(\mu)}\frac{\partial v^{o}_{i}}{\partial x^{o}_{j}(\mu)}C^{o}_{ijkl}(\mu)\frac{\partial u^{o}_{k}(\mu)}{\partial x^{o}_{l}(\mu)}\,d\Omega^{o}(\mu)=\int_{\Gamma^{o}_{3}(\mu)}q^{o}(\mu)e^{o}_{\mathrm{n},i}(\mu)v^{o}_{i}\;d\Gamma^{o}(\mu), \quad \forall v^{o}\in X^{o}(\mu) \tag{3}$$

- Weak form in parameter-independent reference domain Ω
  - Required to map geometric parameter μ_{d} efficiently
  - Enabled by a Jacobian matrix J_{Φ} of a parametric map $\Phi(x;\mu)$

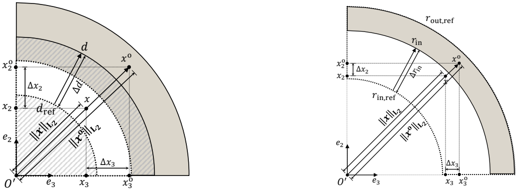
Methods

Geometric parameterization

- Parametric map $\Phi(x;\mu) = x^{o}(x;\mu) = x + \Delta x_{d}(\mu)$
- Geometric parametrization for dent size μ_{d}
  - Transformation ratio: variate the dent size along the ratio $\dfrac{\mu_{d} - d_{ref}}{\|x\|_{L_2}}$
  - Geometric parametrization for each subdomain

Subdomain 1: $x^{o}(x;\mu) = x + \dfrac{\mu_{d} - d_{ref}}{\|x\|_{L_2}} x$

Subdomain 2: $x^{o}(x;\mu) = x + \left(\dfrac{\mu_{d} - d_{ref}}{\|x\|_{L_2}}\right)\left(\dfrac{\|x\|_{L_2} - r_{ref,out}}{r_{ref,in} - r_{ref,out}}\right) x$



(a) Ω_1 (b) Ω_2

Figure 9: Schematic representation of mapping functions

- Weak form in a reference domain

$$\sum_{s=1}^{3} \int_{\Omega_s} \frac{\partial v_i}{\partial x_j} C_{ijkl,s}(x;\mu) \frac{\partial u_k(\mu)}{\partial x_l} d\Omega = \int_{\Gamma_3} q(x;\mu) e_{n,i} v_i \, d\Gamma, \quad \forall v \in X, \qquad (4)$$

where

$$C_{ijkl,s}(x;\mu) = [J_{\Phi_s}^{-1}(x;\mu)]_{jj'} C^{o}_{ij'kl'}(\mu) [J_{\Phi_s}^{-1}(x;\mu)]_{ll'} |J_{\Phi_s}(x;\mu)|,$$

$$q(x;\mu) = q^{o}(\mu) |J_{\Phi_3}(x;\mu) e_{n}|.$$

Methods

Reduced basis approximation

- Dimension reduction modeling technique for parametrized PDE
  - Derive approximate solutions from lower dimensions X^N than finite element solution spaces $X^{\mathcal{N}}$

$$a^N(u^N(\mu), v; \mu) = f^N(v; \mu), \quad \forall v \in X^N$$

  - Achieve a computational efficiency after spending upfront offline computational cost
  - Presume affine parametric dependence to ensure an offline/online decomposition

$$\underbrace{\sum_{q=1}^{Q_a} \theta_a^q(\mu)\, \underbrace{\mathbb{B}^{\mathrm{T}} A_{\mathcal{N}}^q \mathbb{B}}_{\text{offline}}\, u_N(\mu) = \sum_{q=1}^{Q_f} \theta_f^q(\mu)\, \underbrace{\mathbb{B}^{\mathrm{T}} f_{\mathcal{N}}^q}_{\text{offline}}}_{\text{online}}$$

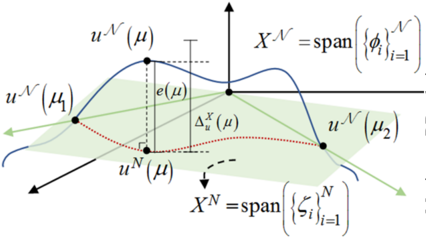


Figure 10: Concept of RB method†

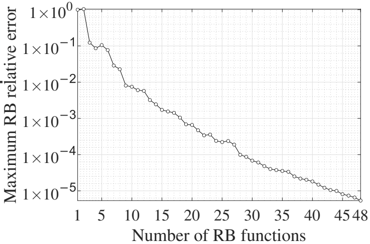


Figure 11: Error convergence for RB training

† Kang, S., & Lee, K. (2021). Real-time, high-fidelity linear elastostatic beam models for engineering education. Journal of Mechanical Science and Technology, 35(8), 3483-3495.

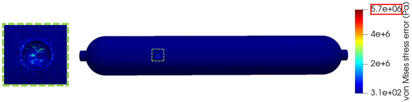
Results

Model verification

- FE vs. RB models
  - Absolute errors of von Mises stress at discrete samples

(a) At minimum parameters = $(E_{\min}, \nu_{\min}, p_{\min}, d_{\min})$

(b) At reference parameters = $(E_{\text{ref}}, \nu_{\text{ref}}, p_{\text{ref}}, d_{\text{ref}})$



(c) At maximum parameters = $(E_{\max}, \nu_{\max}, p_{\max}, d_{\max})$

Figure 12: Absolute errors of von Mises stress between the FE and RB models

- Relative errors for 100 physical parameter samples
  - Validates accuracy of an RB model for physical parameters with a maximum error of less than 1×10^{-5} %

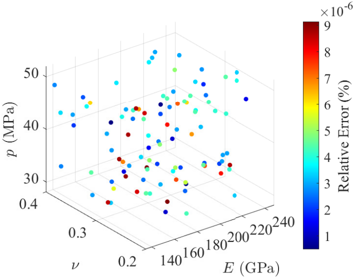


Figure 13: Relative errors of von Mises stress of an RB model for 100 physical parameter samples

Results

Damage scenarios [1/2]

- Scenario 1: vessel with initially identified dent size μ_{d}=0.01 m (number of samples: 10^4)

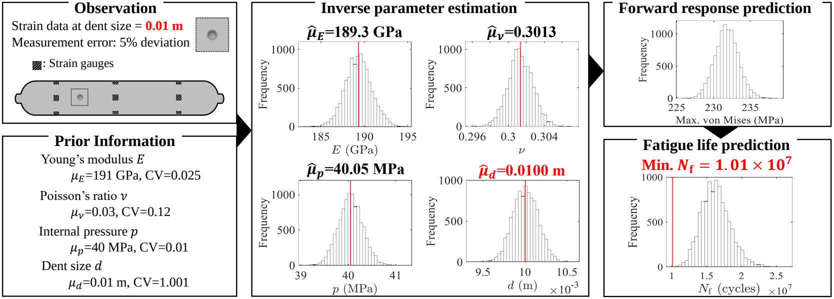


Figure 14: Statistical fatigue life prediction of a damaged vessel for scenario 1

Table 1: Posterior estimates and credible intervals for scenario 1

Parameters	True	Estimated mean	Estimated stdv	95% CI
E [GPa]	191	189.3	1.49	[186.40, 192.25]
ν [-]	0.3000	0.3013	0.0014	[0.2987, 0.3040]
p [MPa]	40	40.05	0.26	[39.55, 40.55]
d [m]	0.0100	0.0100	0.00015	[0.0097, 0.0103]

stdv: standard deviation, CI: credible interval

Results

Damage scenarios [2/2]

- Scenario 2: vessel with enlarged dent size μ_{d}=0.03 m (number of samples: 10^4)

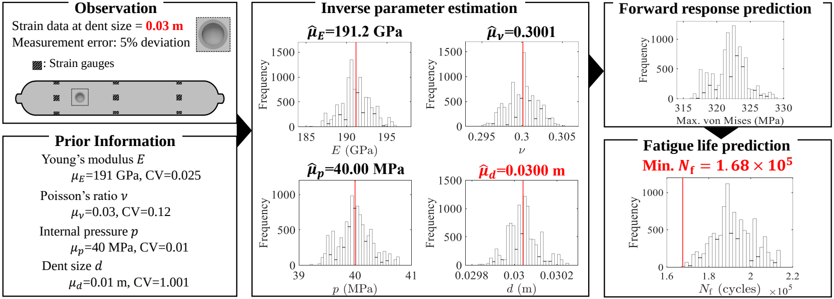


Figure 15: Statistical fatigue life prediction of a damaged vessel for scenario 2

Table 2: Posterior estimates and credible intervals for scenario 1

Parameters	True	Estimated mean	Estimated stdv	95% CI
E [GPa]	191	191.2	1.83	[187.60, 194.76]
ν [-]	0.3000	0.3001	0.0019	[0.2964, 0.3038]
p [MPa]	40	40.00	0.29	[39.43, 40.56]
d [m]	0.0300	0.0300	0.00007	[0.0299, 0.0302]

stdv: standard deviation, CI: credible interval

- Achieved rapid simulations by significantly reducing the dimension
- Offline/online computational time
    - Compared the offline/online computation times of the FE model to that of the RB model

Table 3: Comparison of computation time between FE and RB models

	FE model	RB model	Reduction rate
Dimensions	251,715	48	5.24×10^3
Offline time	-	2 hr 33 min	-
Averaged online time	1 min 44 s	1.98×10^{-4} s	5.24×10^5

- Total evaluation times for statistical fatigue life prediction using digital twin for one scenario

Table 4: Total computational times for statistical fatigue life prediction using digital twin

Phase	FE analysis time	RB analysis time	Reduction rate
Offline training	-	2 hr 33 min	-
Inverse parameter estimation	60 days 1 hr	9.90 s	5.24×10^5
Forward response prediction	12 days	1.98 s	5.24×10^5
Total time	72 days 1 hr	2 hr 33 min	676.20

Summary and conclusions

- Summary

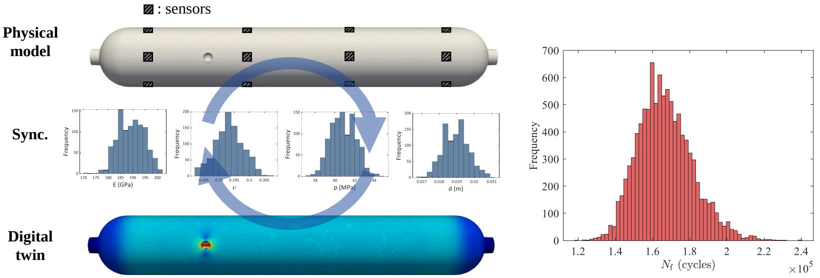


Figure 16: Statistical fatigue life prediction using digital twin

- Conclusions
  - Built a digital twin model with high detection capabilities for the damage status of a physical defect
  - Achieved a rapid diagnosis and prognosis with high accuracy thanks to the RB model
  - Empowered prognostic and health management (PHM) of a damaged pressure vessel by computationally efficient and accurate simulation

Future work

- Overcome challenges of model updating by using a component-based approach
  - Effectively update a model by replacing a component with a defected component after identifying new damage locations

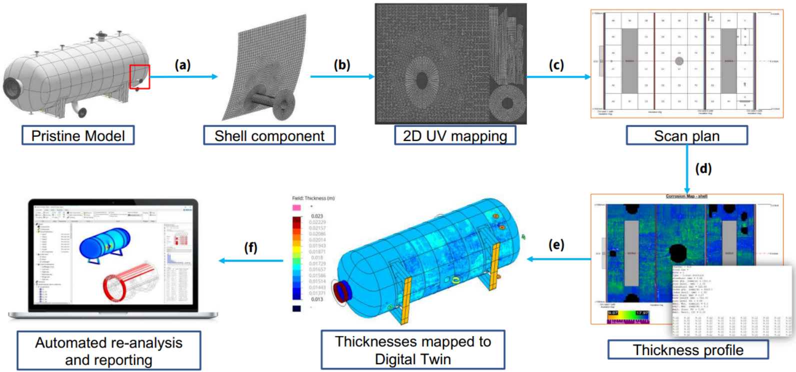


Figure 17: Digital twin of a pressure vessel using a component-based approach†

† Akselos, Case study: digital twin of pressure vessel, https://www.akselos.com/resources-detail/digital-twins-of-pressure-vessels-unlocking-the-full-potential-of-ogtcs-robotic-inspection-joint-industry-project-with-the-oil-and-gas-technology-center