Design Optimization of a Hydrogen Vessel Under Operating Condition Uncertainty via a Parametrized Component-Based Reduced Basis Model

Dayoung Kang

Contents

Motivation

Lightweight and reliable hydrogen storage vessel

- Reliability-based design optimization (RBDO)
  1. achieves the weight reduction for the tube trailer → builds cost-effective transportation system
  2. enables to satisfy the high safety requirements for a vessel under operating condition uncertainty

Figure 1: Storage, transportation, and charging process of a vessel

Rapid yet accurate analysis for a vessel model

- Component-based reduced basis (RB) method
  1. enables to devide a model into simpler components → allows parallel computing
  2. achieves a significant reduction in computational time by virtue of RB method compared to conventional finite element (FE) method

Problem statement

1. formulated the RBDO problem as Fig. 2
2. combined the structural simulation with the RBDO
3. used component-based RB method to evaluate the stress rapidly

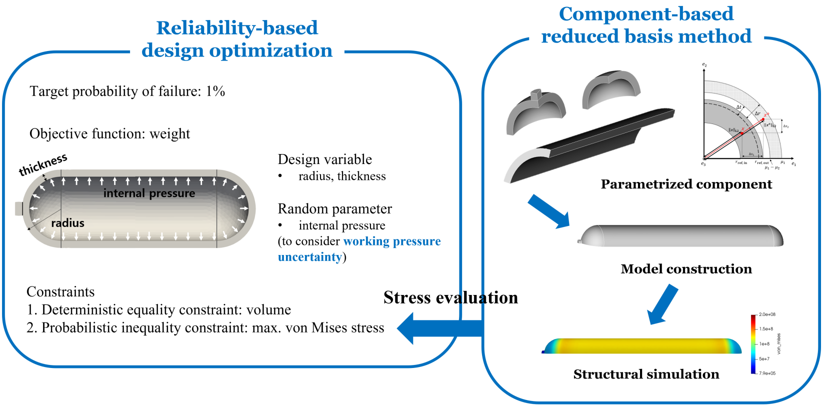

Figure 2: Reliability-based design optimization using component-based reduced basis method

Method
Component-based model

- applied RB approximation to each component $\rightarrow$ static condensation reduced basis element (scRBE) method
- Connected Γ_{p_1} and Γ_{p_2}, Γ_{p_3} and Γ_{p_4} to create a full model
- Defined parametric map of each subdomain Ω_j to parameterize the model for design variables
  Γ_{p_i}: i th port surface $(i = 1, 2, ..., 4)$, Ω_j: jth subdomain $(j = 1, 2, ..., 4)$

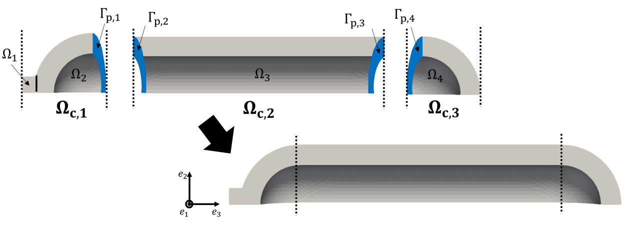

Figure 3: Component-based vessel model

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Method

Geometric parameterization

- Parametric map $M(x;\mu) : \Omega \to \Omega^{o}(\mu)$

$$M(x;\mu) = x^{o}(x;\mu) = x + \Delta x_{r} + \Delta x_{t} \quad (1)$$

Ω: parameter-independent reference domain
$\Omega^{o}(\mu)$: parameter-dependent original domain

- Geometric parametrization
  - Geometric parameters: μ_1: radius, μ_2: thickness

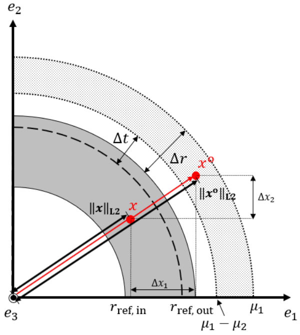


Figure 4: Geometric parametrization of the radius and the thickness

- Radius: mapping for the outer radius $r_{\text{ref, out}}$

$$\Delta x_{r} : \Delta r = x : \|x\|_{L2} \quad (2)$$

where $\Delta r = \mu_1 - r_{\text{ref,out}}$

- Thickness: mapping for the inner radius $r_{\text{ref, in}}$ with outer radius $r_{\text{ref, out}}$ fixed

$$\Delta x_{t} : \Delta t \propto x : r_{\text{ref, in}} \quad (3)$$

where $\Delta t = t_{\text{ref}} - \mu_2$

Method

Geometric parameterization

- Geometric parameterization for radius μ_1

$$\Delta x_{\mathrm{r}} = \frac{\Delta r}{\|x\|_{\mathrm{L2}}} x = \frac{\mu_1 - r_{\text{ref, out}}}{\|x\|_{\mathrm{L2}}} x \tag{4}$$

- Geometric parameterization for thickness μ_2
  - defined a linear interpolation function ϕ

$$\phi(\|x\|_{\mathrm{L2}}) = A\|x\|_{\mathrm{L2}} + B = \begin{cases} 0, & \|x\|_{\mathrm{L2}} = r_{\text{ref, out}} \\ 1, & \|x\|_{\mathrm{L2}} = r_{\text{ref, in}} \end{cases} \tag{5}$$

$$\phi(\|x\|_{\mathrm{L2}}) = \frac{r_{\text{ref, out}} - \|x\|_{\mathrm{L2}}}{t_{\text{ref}}} \tag{6}$$

$$\Delta x_{\mathrm{t}} = \frac{\Delta t}{r_{\text{ref, in}}} \phi(\|x\|_{\mathrm{L2}}) x = \frac{(t_{\text{ref}} - \mu_2)(r_{\text{ref, out}} - \|x\|_{\mathrm{L2}})}{r_{\text{ref, in}} t_{\text{ref}}} x \tag{7}$$

- Geometric parameterization for radius μ_1 and thickness μ_2

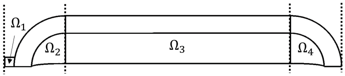
$$\begin{aligned} x^{\mathrm{o}}(x;\mu) &= x + \frac{\mu_1 - r_{\text{ref, out}}}{\|x\|_{\mathrm{L2}}} x + \frac{(t_{\text{ref}} - \mu_2)(r_{\text{ref, out}} - \|x\|_{\mathrm{L2}})}{r_{\text{ref, in}} t_{\text{ref}}} x \\ &= x \left[1 + \frac{\mu_1 - r_{\text{ref, out}}}{\|x\|_{\mathrm{L2}}} + \frac{(t_{\text{ref}} - \mu_2)(r_{\text{ref, out}} - \|x\|_{\mathrm{L2}})}{r_{\text{ref, in}} t_{\text{ref}}} \right] \end{aligned} \tag{8}$$

Method

Geometric parameterization

Table 1: Parametric maps of subdomains

Subdomain	Parametric map $M(x;\mu)$
Ω_1	$x_1^o(x;\mu) = x_1 + x_1\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_1}}\right]$ $x_2^o(x;\mu) = x_2 + x_2\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_1}}\right]$ $x_3^o(x;\mu) = x_3 + x_3\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_1}}\right]$
Ω_2	$x_1^o(x;\mu) = x_1 + x_1\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_2}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_2}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_2^o(x;\mu) = x_2 + x_2\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_2}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_2}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_3^o(x;\mu) = x_3 + x_3\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_2}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_2}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$
Ω_3	$x_1^o(x;\mu) = x_1 + x_1\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_3}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_3}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_2^o(x;\mu) = x_2 + x_2\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_3}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_3}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_3^o(x;\mu) = x_3$
Ω_4	$x_1^o(x;\mu) = x_1 + x_1\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_4}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_4}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_2^o(x;\mu) = x_2 + x_2\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_4}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_4}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$ $x_3^o(x;\mu) = x_3 + x_3\left[\frac{\mu_1 - r_{\text{ref,out}}}{\lVert x\rVert_{L2,\Omega_4}} + \frac{(t_{\text{ref}} - \mu_2)\left(r_{\text{ref,out}} - \lVert x\rVert_{L2,\Omega_4}\right)}{r_{\text{ref,in}}\, t_{\text{ref}}}\right]$



Method

Linear elasticity problem

- Strong form

$$\frac{\partial}{\partial x_j^o}\left(C_{ijkl}^o \frac{\partial u_k^o(\mu)}{\partial x_l^o}\right) = 0, \quad \text{in} \quad \Omega^o(\mu) \tag{9}$$

- Boundary conditions

$$\begin{aligned} u^o &= 0 \quad \text{on} \quad \Gamma_1^o \\ u_1^o &= 0 \quad \text{on} \quad \Gamma_2^o \\ u_2^o &= 0 \quad \text{on} \quad \Gamma_3^o \\ qe_{i,\mathrm{n}}^o & \quad\quad \text{on} \quad \Gamma_4^o \end{aligned} \tag{10}$$

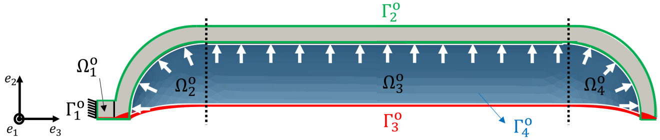


Figure 5: Boundary conditions of the vessel model

- Weak form

$$\int_{\Omega^o(\mu)} \frac{\partial v_i^o}{\partial x_j^o} C_{ijkl}^o(\mu) \frac{\partial u_k^o(\mu)}{\partial x_l^o} d\Omega = \int_{\Gamma_4^o(\mu)} v_i^o q e_{i,\mathrm{n}}^o d\Gamma \tag{11}$$

Method

Reduced basis (RB) method

- Mapping from $\Omega^{o}(\mu)$ to Ω
  - requires inverse transformation of the geometric map $M^{-1}(x;\mu)$.
- Domain transformation
  - Define jacobian matrix $\mathbb{J}_M$ of a $M(x;\mu)$.

$$(\mathbb{J}_M)_{pq} = \frac{\partial x_p^{o}}{\partial x_q}(x) = \frac{\partial M_p(x;\mu)}{\partial x_q}(x) \tag{12}$$

  - Define inverse matrix of $\mathbb{J}_M$.

$$(\mathbb{J}_{M^{-1}})_{pq} = \frac{\partial x_p}{\partial x_q^{o}}(x^{o}) = \frac{\partial M_p^{-1}(x^{o};\mu)}{\partial x_q^{o}}(x^{o}) \tag{13}$$

- Weak form in bilinear and linear forms for a reference domain

$$a(u(\mu), v;\mu) = f(v;\mu) \tag{14}$$

where

$$\begin{aligned} a(u(\mu), v;\mu) &= \int_{\Omega} \frac{\partial v_i}{\partial x_m} C_{imkn}(\mu) \frac{\partial u_k}{\partial x_n} d\Omega \\ f(v;\mu) &= \int_{\Gamma_4} v_i q e_{i,\mathrm{n}} \left|(\mathbb{J}_M) e_{\mathrm{t}}\right| d\Gamma \end{aligned} \tag{15}$$

- Effective constitutive tensor C_{imkn}

$$C_{imkn} = (\mathbb{J}_{M^{-1}})_{mj} C_{ijkl}^{o}(\mu) (\mathbb{J}_{M^{-1}})_{itnl} \left|(\mathbb{J}_M)\right| \tag{16}$$

Method

Reduced basis (RB) method

- FE linear system
  - FE solution
$$u(\mu) = \sum_{i=1}^{\mathcal{N}} u^i_{\mathcal{N}}(\mu)\xi_i \tag{17}$$
  - Substituting eq. 17 to $a(u(\mu), v; \mu) = f(v; \mu)$,
$$\mathbb{A}_{\mathcal{N}} u_{\mathcal{N}} = \mathbb{F}_{\mathcal{N}} \tag{18}$$
where $(\mathbb{A}_{\mathcal{N}})_{ij} = a(\xi_i, \xi_j; \mu), \quad (\mathbb{F}_{\mathcal{N}})_i = f(\xi_i; \mu), \quad 1 \leq i, j \leq \mathcal{N}$
- Reduced basis (RB) method
  - Dimension reduction modeling technique for parameterized PDE
$$\mathbb{A}_{\mathcal{N}} u_{\mathcal{N}} = \mathbb{F}_{\mathcal{N}} \quad \Rightarrow \quad \mathbb{A}_N u_N = \mathbb{F}_N \quad \text{where } N \ll \mathcal{N} \tag{19}$$

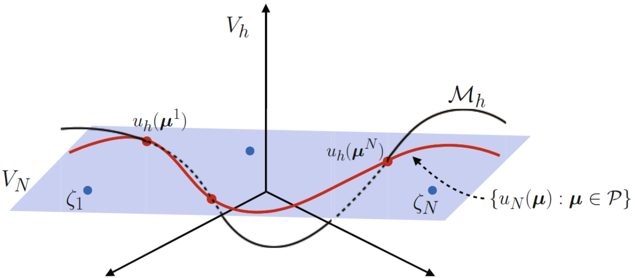


Figure 6: Schematic illustration for the reduced basis method

Method

Static condensation reduced basis element (scRBE) method

- scRBE method
  - A component-based approach combined with RB method
  - uses RB method to component interior to reduce the solution dimension
- Component-based FE system

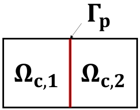


Figure 7: Two components system with an in-between port

$$\begin{bmatrix} A_{\Gamma_p} & A^T_{\Gamma_p,\Omega_{c,1}} & A^T_{\Gamma_p,\Omega_{c,2}} \\ A_{\Gamma_p,\Omega_{c,1}} & A_{\Omega_{c,1}} & 0 \\ A_{\Gamma_p,\Omega_{c,2}} & 0 & A_{\Omega_{c,2}} \end{bmatrix} \begin{bmatrix} u_{\Gamma_p} \\ u_{\Omega_{c,1}} \\ u_{\Omega_{c,2}} \end{bmatrix} = \begin{bmatrix} f_{\Gamma_p} \\ f_{\Omega_{c,1}} \\ f_{\Omega_{c,2}} \end{bmatrix}$$

Represent $u_{\Omega_{c,1}}$ and $u_{\Omega_{c,2}}$ in terms of u_{Γ_p}

$$\underbrace{(A_{\Gamma_p} - A^T_{\Gamma_p,\Omega_{c,1}} A^{-1}_{\Omega_{c,1}} A_{\Gamma_p,\Omega_{c,1}} - A^T_{\Gamma_p,\Omega_{c,2}} A^{-1}_{\Omega_{c,2}} A_{\Gamma_p,\Omega_{c,2}})}_{\mathbb{A}_{sc}} u_{\Gamma_p} = \underbrace{f_{\Gamma_p} - A^T_{\Gamma_p,\Omega_{c,1}} A^{-1}_{\Omega_{c,1}} f_{\Omega_{c,1}} - A^T_{\Gamma_p,\Omega_{c,2}} A^{-1}_{\Omega_{c,2}} f_{\Omega_{c,2}}}_{\mathbb{F}_{sc}} \quad (20)$$

$$\mathbb{A}_{sc} u_{\Gamma_p} = \mathbb{F}_{sc} \quad (21)$$

Method
Static condensation reduced basis element (scRBE) method

- Component-based FE system $\underbrace{\mathbb{A}_{\mathrm{sc}}}_{\mathcal{N}_{\mathrm{p}}\times\mathcal{N}_{\mathrm{p}}} \underbrace{u_{\Gamma_{\mathrm{p}}}}_{\mathcal{N}_{\mathrm{p}}\times 1} = \underbrace{\mathbb{F}_{\mathrm{sc}}}_{\mathcal{N}_{\mathrm{p}}\times 1}$

$$\begin{aligned} \mathbb{A}_{\mathrm{sc}} &= A_{\Gamma_{\mathrm{p}}} - A^{\mathrm{T}}_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},1}} \underbrace{A^{-1}_{\Omega_{\mathrm{c},1}} A_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},1}}}_{\text{FE bubble } b^1_{\mathbb{A},k}} - A^{\mathrm{T}}_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},2}} \underbrace{A^{-1}_{\Omega_{\mathrm{c},2}} A_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},2}}}_{\text{FE buble } b^2_{\mathbb{A},k}} \\ \mathbb{F}_{\mathrm{sc}} &= f_{\Gamma_{\mathrm{p}}} - A^{\mathrm{T}}_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},1}} \underbrace{A^{-1}_{\Omega_{\mathrm{c},1}} f_{\Omega_{\mathrm{c},1}}}_{\text{FE bubble } b^1_{\mathbb{F}}} - A^{\mathrm{T}}_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},2}} \underbrace{A^{-1}_{\Omega_{\mathrm{c},2}} f_{\Omega_{\mathrm{c},2}}}_{\text{FE bubble } b^2_{\mathbb{F}}} \end{aligned} \tag{22}$$

where

$$\begin{aligned} A^{-1}_{\Omega_{\mathrm{c},i}} A_{\Gamma_{\mathrm{p}},\Omega_{\mathrm{c},i}} &= A^{-1}_{\Omega_{\mathrm{c},1}} [a^i_1, \cdots a^i_{\mathcal{N}_{\mathrm{p}}}] = [b^i_{\mathbb{A},1}, \cdots b^i_{\mathbb{A},\mathcal{N}_{\mathrm{p}}}] \\ A^{-1}_{\Omega_{\mathrm{c},i}} f_{\Omega_{\mathrm{c},i}} &= b^i_{\mathbb{F}} \end{aligned} \tag{23}$$

- Component-based RB system (scRBE system) $\tilde{\mathbb{A}}_{\mathrm{sc}} \tilde{u}_{\Gamma_{\mathrm{p}}} = \tilde{\mathbb{F}}_{\mathrm{sc}}$
  - Apply RB method to FE bubbles

$$\begin{aligned} b^i_{\mathbb{A},k} &\rightarrow B^i_{\mathbb{A},k} \tilde{b}^i_{\mathbb{A},k} \\ b^i_{\mathbb{F}} &\rightarrow B^i_{\mathbb{F}} \tilde{b}^i_{\mathbb{F}} \end{aligned} \tag{24}$$

$\Rightarrow$ Enable a tremendous speedup compared to component-based FE system

Method
Reliability-based design optimization (RBDO)

Reliability-based design optimization (RBDO) formulation

$$
\begin{aligned}
&\text{find} && r, t \in \mathbb{R} \\
&\text{minimize} && \text{weight}(r, t), \\
&\text{subject to} && \text{volume}(r, t) - V = 0, \\
& && P[G(r,t,p) > 0] \le P_{\mathrm{F}}^{\mathrm{Target}} = 1\% \\
& && r_{\mathrm{LB}} \le r \le r_{\mathrm{UB}}, \quad t_{\mathrm{LB}} \le t \le t_{\mathrm{UB}}.
\end{aligned}
$$

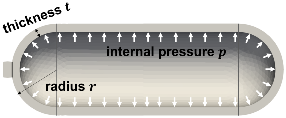


Figure 8: Design variables and parameter

- Deterministic design variables r, t
- Probabilistic random variable p
  - Random variable that follows normal distribution
  - Mean: operating pressure 40 MPa
  - Coefficient of variation: 5%
- Limit state function G

$$G(r,t,p) = \sigma_{\max}(r,t,p) - \sigma_{\mathrm{a}} \tag{25}$$

Method

Reliability-based design optimization (RBDO)

- Probability of failure P_F

$$P_F = \int_{G(y)>0} f_Y(y)dy \tag{26}$$

f_Y: probability density function of random variable Y

- Approximation to the probability of failure
  - Used first-order reliability method to approximate P_F
  - Transform random variable and limit state function from non-normal distribution space to normal distribution space using Rosenblatt transformation

$$P_F \approx \Phi(-\beta) \tag{27}$$

where $\beta = \|\mathbf{u}^*\|$

Φ: cumulative distribution function of the standard normal distribution
β: reliability index
$\mathbf{u}^*$: most probable point (MPP)

  - Used performance measure approach (PMA) to search MPP
    - Required to evaluate $\beta = \|\mathbf{u}^*\|$

Method

Reliability-based design optimization (RBDO)

- Performance measure apporach (PMA)
  - One of the methods to search MPP
  - Fix β to target value β^{T}
  - Higher convergence rate than the conventional reliability index approach (RIA)

$$
\begin{aligned}
&\text{for a given design} \quad r, t \in \mathbb{R}, \\
&\text{find} \qquad \mathbf{u}^*, \\
&\text{minimize} \quad G(r, t, p_{\mathrm{u}}), \\
&\text{subject to} \quad \|\mathbf{u}\| = \beta^{\mathrm{T}}.
\end{aligned} \tag{28}
$$

p_{u}: random variable p_{d} after Rosenblatt transformation

Results

Reliability-based design optimization (RBDO)

- RBDO result

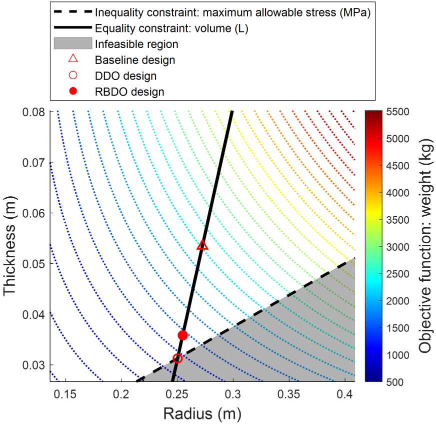


Figure 9: Optimization results and comparison of design points

DDO: Deterministic design optimization
RBDO: Reliability-based design optimization
N_e: Number of function evaluations
P_F: Probability of failure

Table 2: Optimization results summary

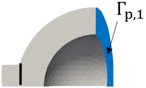
		Baseline	DDO	RBDO
Cost	Weight (kg)	2300	1276	1479
	Reduction (%)	-	44.5	35.7
Design variables	Radius (m)	0.272	0.250	0.255
	Thickness (m)	0.053	0.031	0.036
Maximum stress (MPa)		197.702	298.667	267.544
N_e		-	24	156
P_F (%)		0.000	51.000	1.001

Results
Model dimensions

- FE vs. scRBE dimensions
  - Example: $\Omega_{c,1}$

Table 3: FE and scRBE dimensions for the component $\Omega_{c,1}$

			FE dimension	scRBE dimension	Reduction rate
Port	$\Gamma_{p,1}$		1281	13	98.5
Port dependent bubble space	$\Gamma_{p,1}$	DOF 1	23550	15	1570
		DOF 2	23550	15	1570
		DOF 3	23550	15	1570
		DOF 4	23550	14	1682.1
		DOF 5	23550	14	1682.1
		DOF 6	23550	14	1682.1
		DOF 7	23550	15	1570
		DOF 8	23550	18	1308.3
		DOF 9	23550	14	1682.1
		DOF 10	23550	14	1682.1
		DOF 11	23550	18	1308.3
		DOF 12	23550	19	1239.5
		DOF 13	23550	14	1682.1
		...	...	-	
		DOF 1281	23550		
Port independent bubble space			70650	14	5046.4



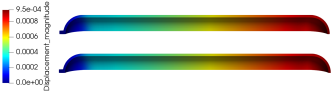
$\Omega_{c,1}$

Results

Solutions

- FE vs. scRBE solutions
  - Displacement magnitudes and absoulute errors

(a) Baseline design

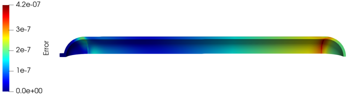


(b) DDO design

(c) RBDO design

Figure 10: Displacement magnitudes of the FE (top) and the scRBE models (bottom)

(a) Baseline design



(b) DDO design

(c) RBDO design

Figure 11: Displacement error norm between the FE and the scRBE models

Results

Computational times

- FE vs. scRBE solutions
  - Single evaluation times of FE and scRBE models

Table 4: Computaitonal times of the FE and online scRBE models

Design	FE anlysis time (s)	scRBE analysis time (s)	Reduction rate
DDO	76.8814	0.00716	10737.6
RBDO	76.5393	0.00722	10601.0
Baseline	76.2145	0.00727	10483.4

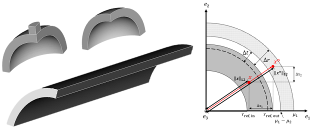
  - Total evaluation times of FE and scRBE models

Table 5: Computational times of optimization with the FE and online scRBE models

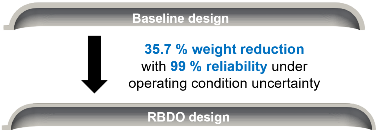
Optimization type	FE anlysis time (s)	scRBE analysis time (s)	Reduction rate
DDO	1837.0816	0.1732	10606.7
RBDO	11941.0304	1.1258	

Summary and conclusions

- Summary



Parametrized component-based reduced basis model



Reliability-based design optimization

- Conclusions
  - Performed RBDO in an accurate and computationally efficient manner with the aid of the scRBE method
  - Contribute to cut down the time to reach an optimized design by realizing the real-time simulations

Future work

- Build digital twin model for fatigue diagnosis